Violation of Lorentz Invariance and neutral component of UHECR

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Abstract

The observed clustering of ultra-high energy cosmic rays suggests the existence of a neutral component. The models with violation of Lorentz invariance may explain this component by neutrons becoming stable above some threshold energy E_0. The protons, in turn, may become unstable above some energy $E_1 > E_0$. We calculate the dependence of the threshold energies E_0 and E_1 on the parameters of the model and find $E_1/E_0 \gtrsim 1.5$. We argue that the characteristic threshold behavior of charged and neutral components may be used as the specific signature of models with violation of Lorentz invariance. The existence of the neutron stability threshold E_0 can be investigated with already existing data.

1 Introduction

Ultra-high energy cosmic ray (UHECR) experiments are believed to be one of the promising places where physics beyond the Standard Model may be discovered. Until recently this expectation was based on a number of arguments of which the non-observation [1] of the GZK cutoff [2] is most acknowledged. The case is strengthened by the absence of nearby candidate sources in the directions of the observed UHECR, large-scale isotropy of arrival directions and generic difficulties with acceleration of particles to energies of order 10^{20} eV.

The apparent absence of the GZK cutoff by itself can be explained within the conventional physics. For instance, several models have been proposed

which assume isotropization of arrival directions in strong magnetic fields and attribute the observed UHECR to neutron stars of our Galaxy [3], Virgo cluster [4] or the nearby radio-galaxy Cen A [5]. Regardless of a particular model, assuming hard injection spectrum in combination with local over-density of sources at scales ~ 50 Mpc makes the discrepancy between the observed and predicted spectra significantly smaller [6, 7].

A new ingredient in this problem is the small scale anisotropy of UHECR. The study of arrival directions of UHECR reveals the existence of clusters of events [8, 9, 10]. The analysis based on angular correlation function shows that the typical size of clusters is comparable with the experimental angular resolution [11, 12]. A most natural interpretation of this result is the existence of point-like sources of UHECR whose image is smeared by errors in determination of UHECR arrival directions.

If this interpretation is believed, it has several important consequences. First, it excludes the models which assume diffuse propagation of UHECR in strong magnetic fields. Second, it makes very unlikely the scenarios which explain the absence of the GZK cutoff by assuming hard spectrum and local overdensity of sources. Indeed, in the latter case the arguments based on statistics of clustering impose unrealistic bounds on the local density of sources [13, 14]. Finally, the observed clustering suggests the existence of neutral primary particles, since otherwise they would be deflected in the Galactic magnetic fields and clusters would not be so tight.

Even stronger argument in favor of neutral particles follows from the correlation of arrival directions with BL Lacertae objects (BL Lac) [15]. BL Lacs are located at cosmological distances. For instance, two of the BL Lacs which coincide with triplets of UHECR are both at ~ 600 Mpc. If primary particles of the triplet events indeed came from BL Lacs, they must be neutral; moreover, they must be able to propagate over cosmological distances without substantial attenuation.

The existence of neutral particles imposes further constraints on possible models of UHECR. Within the Standard Model, there are two stable neutral particles, photon and neutrino. The photon attenuation length due to e^+e^- pair production on infrared background is of order $10-20$ Mpc in the energy range $10^{19}-10^{20}$ eV [16], so photons cannot explain the observed events unless the infrared background is extremely low and/or their initial energies are extremely high.

Neutrinos by themselves cannot be particles which initiate airshowers if standard neutrino cross sections are assumed. They have to be converted into

2

hadrons and photons through interactions with primordial neutrino background [17, 18, 19]. Photons obtained in this way could account for the observed tight clusters. There are generic problems with this scenario [20]. First, the smallness of neutrino cross sections implies large neutrino flux. Second, the production of neutrinos by accelerated protons requires significantly higher proton energy, while even energies of order 10^{20} eV are difficult to achieve in most acceleration sites. If these difficulties can be overcome, neutrino may be an appealing candidate.

While the situation with neutrinos is unclear, it is worth considering other possibilities. Outside of the Standard Model, several candidates for neutral primary particles have been proposed. One of them is a light SUSY hadron [21, 22] which can be, e.g., a uds-gluino bound state or gluon-gluino bound state. Another possible candidate suggested recently is a light sgoldstino [23]. The models of this type have an advantage of explaining the neutral component and the absence of the GZK cutoff at the same time. The disadvantage is that primary particles have to be produced by accelerated protons, which leads to losses in both flux and energy. As a result, already very tight requirements on acceleration energy and efficiency become even tighter.

An attractive model which shares the same advantages but does not have the latter drawback has been proposed in Ref. [24]. It is based on possible violation of Lorentz invariance at high energies [24, 25] (see, e.g., [26] for further discussion of Lorentz invariance violation in the context of UHECR). Models of this type can be constructed without giving up the main principles of quantum field theory in the framework of the brane world scenario [27, 28, 29]. Under rather general assumptions, violation of Lorentz invariance can be described phenomenologically by introducing different maximum velocities for different particles. As noted in Ref. [24], these parameters can be arranged in such a way that the main cause of the GZK cutoff, the pion photoproduction process, is not operative. At the same value of parameters, neutron may become heavier than proton at sufficiently high energy, and therefore stable. Thus, it can serve as a neutral primary particle which is indistinguishable from proton in other respects. Moreover, protons (which are stable at low energies) may be accelerated in a usual way. Once they reach the threshold energy and become unstable, they decay into neutrons by "β-decay" with no loss in the flux and practically no loss in energy. Another way to produce UHE neutrons is in collisions of UHE protons with synchrotron photons in the acceleration site.

How the above models can be distinguished? A clear signature of the

neutrino model is the direct detection of ultra-high energy neutrinos, whose flux is expected to be in the range of sensitivity of the Pierre Auger and Telescope Array experiments [30]. An indirect argument in favor of neutrino models would be identification of the neutral component as photons. On the contrary, the hadronic neutral component speaks in favor of light SUSY hadron models or violation of Lorentz invariance.

The purpose of this paper is to point out the signature which can discriminate between the latter two possibilities. It is based on the fact that within the model of Ref. [24] the behavior of proton and neutron masses with energy follows a certain pattern: with increasing energy, first the neutron becomes stable at some energy E_0, and then proton becomes unstable at some higher energy $E_1 > E_0$. In other words, there is a window in which both neutron and proton are stable, while at higher energies proton becomes unstable. Thus, in the model of Ref. [24], one generically expects that there is only charged component at $E < E_0$, both charged and neutral components at $E_0 < E < E_1$, and only neutral component at $E > E_1$. The appearance and disappearance of components is a threshold effect and is, therefore, a step-like function of energy. This step-like behavior is the specific signature of models with violation of Lorentz invariance.

2 Kinematics of the nucleon decays

Let us now calculate the width of the energy window in which the two components coexist. Following Ref. [24] consider the model characterized by the maximum attainable velocities of neutron, proton, electron and neutrino equal to c_n, c_p, c_e and c_ν, respectively. Without loss of generality one may set

$$c_n = 1 .$$

Certainly, c_p, c_e and c_ν should be very close to 1 as well. The differences between them are at the level of $\lesssim 10^{-20}$ [24]. As it was shown in Ref. [24], the neutron is stable at high energies only if

$$c_p, \ c_e, \ c_\nu \geq 1 .$$

Moreover, if $c_p \neq 1$ then proton becomes unstable at high enough energy. We will show that the ratio E_0/E_1 is determined by the parameter

$$\alpha \equiv \frac{c_e^2 - c_n^2}{c_p^2 - c_n^2}, \tag{1}$$

while the absolute magnitudes of both E_0 and E_1 are proportional to ϵ^{-1}, where

$$\epsilon \equiv (c_p^2 - c_n^2)^{1/2}. \tag{2}$$

Following the formalism of Ref. [24], let us find the energy E_0 above which the neutron is stable, as a function of the parameters c_p, c_e and c_ν. In order to do this one has to find the minimum possible energy $E_{\mathrm{mn}}(p_n)$ of the potential decay products, i.e. proton, electron and neutrino, at fixed total momentum p_n. The condition of stability of the neutron with the momentum p_n and energy $E_n = \sqrt{p_n^2 + m_n^2}$ has the form

$$\sqrt{p_n^2 + m_n^2} \leq E_{\mathrm{mn}}(p_n) . \tag{3}$$

As it was shown in Ref. [24], the equality in Eq. (3) may hold only at one value of the momentum p_n. Consequently, the energy E_0 is equal to

$$E_0 = \sqrt{p_0^2 + m_n^2} , \tag{4}$$

where p_0 is a solution to the equation

$$\sqrt{p_0^2 + m_n^2} = E_{mn}(p_0) . \tag{5}$$

Clearly, the minimum total energy of the proton, electron and neutrino with the fixed total momentum is achieved when the momenta of all three particles are collinear. Thus, in order to calculate $E_{mn}(p_0)$ one should find the minimum value of the following function

$$\mathcal{E}_n(p_p, p_e, p_\nu) = c_p\sqrt{p_p^2 + m_p^2 c_p^2} + c_e\sqrt{p_e^2 + m_e^2 c_e^2} + c_\nu p_\nu \tag{6}$$

of three positive variables p_p, p_e and p_ν subject to the constraint

$$p_0 = p_p + p_e + p_\nu . \tag{7}$$

This minimum may be either an extremum of the function $\mathcal{E}_n(p_p, p_e, p_\nu)$, or belong to the boundary of the triangular region determined by the constraint (7) and positivity conditions

$$p_p, \ p_e, \ p_\nu \geq 0 .$$

As is shown in the Appendix, the minimum is achieved at zero neutrino energy and momentum,

$$p_\nu = 0 \ .$$

Therefore, in order to find the energy at which neutron becomes stable one should study the kinematics of two-body decay $n \to pe$. As a consequence, E_0 does not depend on the neutrino velocity c_ν. The problem reduces to the following system of equations

$$p_0 = p_e + p_p \tag{8}$$

$$\frac{c_e p_e}{\sqrt{p_e^2 + m_e^2 c_e^2}} = \frac{c_p p_p}{\sqrt{p_p^2 + m_p^2 c_p^2}} \tag{9}$$

$$\sqrt{p_0^2 + m_n^2} = c_e \sqrt{p_e^2 + m_e^2 c_e^2} + c_p \sqrt{p_p^2 + m_p^2 c_p^2} \ . \tag{10}$$

Here eq.(8) is the momentum conservation condition (7) with $p_\nu = 0$, eq.(9) is the extremality condition for the function $\mathcal{E}(p_p, p_e, 0)$ and eq.(9) is the stability condition (5). It is straightforward to solve this system analytically in the case $c_e = c_p$ with the result

$$E_0 \approx \sqrt{\frac{m_n^2 - (m_p + m_e)^2}{c_p^2 - 1}} \sim 3.85 \times 10^{19} \left(\frac{10^{-12}}{\epsilon} \right) \text{ eV}, \tag{11}$$

in agreement with Ref. [24].

In the case $c_p \neq c_e$ it is convenient to express the velocities in terms of the parameters α and ϵ according to

$$c_p^2 = 1 + \epsilon^2, \ c_e^2 = 1 + \alpha \epsilon^2,$$

and rescale the momenta

$$p_i = \frac{q_i}{\epsilon}.$$

We assume ϵ to be very small, but make no assumptions about the value of α. Expanding eqs.(8)-(10) to the lowest non-trivial order in ϵ we obtain a system of polynomial equations for three unknowns q_0, q_p, q_e, which depends on one parameter α. This system can be solved for q_0 numerically. The threshold energy is determined by the relation
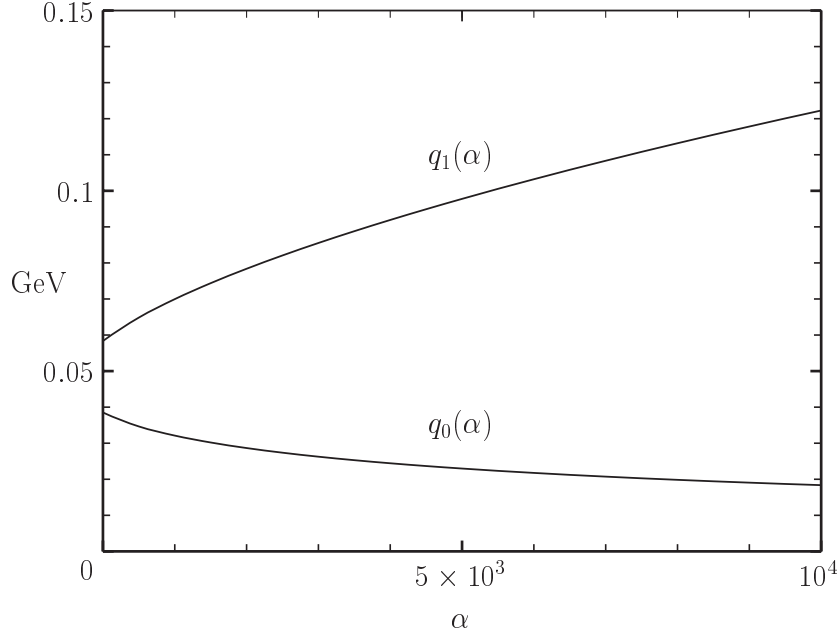
$$E_0 = \frac{q_0(\alpha)}{\epsilon}, \tag{12}$$

Figure 1: The dependence of the rescaled threshold momenta $q_0 \approx \epsilon E_0$ and $q_1 \approx \epsilon E_1$ on the parameter α.

where the difference between E_0 and p_0 has been neglected. The numerical solution for q_0 as a function of α is plotted in Fig. 1.

Let us now find the threshold energy E_1 above which the decay $p \to ne\bar{\nu}$ is kinematicaly allowed. We follow the similar procedure as in the case of neutron, *i.e.*, find the threshold momentum p_1 from the equation

$$c_p\sqrt{p_1^2 + m_p^2 c_p^2} = E_{mp}(p_1) \, , \tag{13}$$

where $E_{mp}(p_p)$ is the minimum total energy of a neutron, electron and neutrino with the fixed total momentum p_p. This energy is obtained by the minimization of the function

$$\mathcal{E}_p(p_n, p_e, p_\nu) = \sqrt{p_n^2 + m_n^2} + c_e\sqrt{p_e^2 + m_e^2 c_e^2} + c_\nu p_\nu \tag{14}$$

7

under the constraints

$$p_n + p_e + p_\nu = p_p, \qquad p_n, p_e, p_\nu \geq 0 \ . \qquad (15)$$

As in the case of a neutron one may show that the minimum is reached at $p_\nu = 0$ and therefore it is sufficient to study the kinematics of the two-body decay $p \to ne$ when both momenta p_p and p_e are non-zero. The system of equations determining the threshold momentum p_1 has the following form (cf. Eqs. (8)-(10))

$$p_1 = p_e + p_n \qquad (16)$$

$$\frac{c_e p_e}{\sqrt{p_e^2 + m_e^2 c_e^2}} = \frac{p_n}{\sqrt{p_n^2 + m_n^2}} \qquad (17)$$

$$c_p \sqrt{p_1^2 + c_p^2 m_p^2} = c_e \sqrt{p_e^2 + c_e^2 m_e^2} + \sqrt{p_n^2 + m_n^2} \ . \qquad (18)$$

Again, the solution is straightforward to find in the case $c_p = c_e$ (cf. Ref. [24]),

$$E_1 \approx \frac{\sqrt{m_n^2 - (m_p - m_e)^2}}{\epsilon} \sim 5.83 \times 10^{19} \left(\frac{10^{-12}}{\epsilon} \right) \text{eV}, \qquad (19)$$

At $c_p \neq c_e$, the solution can be obtained numerically. The resulting function $q_1(\alpha)$ is plotted in Fig. 1. At small ϵ, the threshold energy E_1 is related to $q_1(\alpha)$ by

$$E_1 = \frac{q_1(\alpha)}{\epsilon}. \qquad (20)$$

Comparing Eqs.(11) and (19) one finds that the ratio $E_1/E_0 \sim 1.5$ at $c_p = c_e$ (i.e., at $\alpha = 1$). It grows with α, as is shown in Fig. 2. The dependence becomes nearly linear at the values $\alpha \gtrsim m_p/m_e \sim 2000$.

3   Discussion

As we have seen in Sect. 2, the threshold energies of neutron stability and proton instability, E_0 and E_1, depend on two parameters ϵ and α as defined by eqs. (12) and (20). The parameter ϵ sets an overall scale of E_0 and E_1, while the parameter α uniquely determines the ratio E_1/E_0. This ratio is always larger than ~ 1.5.

In order to be of phenomenological interest, the neutron stability threshold E_0 must be low enough. If clustering in Yakutsk data [11] is attributed to
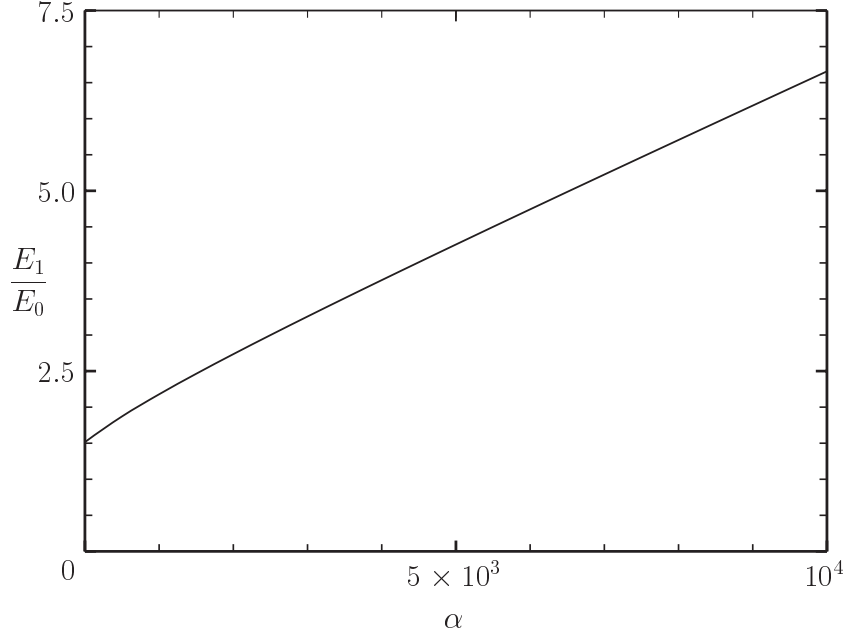
Figure 2: The ratio of threshold energies E_1/E_0 as the function of the parameter α.

neutrons, the existence of triplet with energies $(2.5; 2.8; 3.4) \times 10^{19}$ eV implies $E_0 < 2.5 \times 10^{19}$ eV. At $\alpha \ll m_p/m_e$ this requires in turn $\epsilon > 1.5 \times 10^{-12}$, or $c_p^2 - c_n^2 > 2 \times 10^{-24}$ (this constraint is somewhat weaker for large values of α, see fig.1). This constraint does not contradict the existing upper bounds [24] on $c_p^2 - 1$ and $c_n^2 - 1$. Interestingly, the threshold at E_0 may, in principle, be detected (or ruled out) with already existing experimental data at low energies $E < 4 \times 10^{19}$ eV. For this analysis the key issue is the angular resolution of the experiment, so the AGASA data seems the most suitable choice. Note that the improvement of upper limits on $c_p^2 - c_n^2$ may rule out the models which explain the neutral component of UHECR by the violation of the Lorentz invariance.

The threshold of proton instability E_1 is more difficult to detect since this

9

would require good identification of charged component. In principle, this can be done by making use of the deflections in the Galactic and extragalactic magnetic fields. Two different situations should be distinguished. If at energies smaller than E_1 the extragalactic fields cause random deflections by angles larger than the typical distance between the sources, the charged component would look like a uniform background. It will be difficult, if possible at all, to separate such a component from unresolved sources.

On the contrary, if random deflections are smaller than typical separation between the sources, one would see "halos" of the events formed by charged primary particles around clusters of the events with neutral primaries. If correlations with BL Lacs at large angles [15] are believed, current data favor the latter situation. If so, with enough statistics one will be able to identify the charged component reliably. Even better situation is possible if random deflections are negligible. In that case not only the charged particles can be identified, but their charges and the structure of magnetic fields can, in principle, be determined.

It is worth noting that models with exotic neutral primaries also generically predict the flux of charged particles, the direct protons from the source. The difference with the previous case is that in these models the proton component has a conventional GZK cutoff which occurs at fixed and calculable energy E_{GZK}, while the proton instability threshold E_1 is, in general, different. If by chance $E_1 \simeq E_{\mathrm{GZK}}$, the discrimination between the two models would be very difficult.

To summarize, we argue that the characteristic threshold behavior of charged and neutral components in models with violation of Lorentz invariance can be used as their experimental signature. Upper bound on the neutron stability threshold E_0 can be obtained from already existing data. the proton stability threshold E_1 may be seen by the Pierre Auger experiment [31] where about 1000 events in the energy range $10^{19} - 10^{20}$ eV are expected in a year.

Acknowledgements

Appendix. Minimization of $\mathcal{E}_n(p_p, p_e, p_\nu)$

It is straightforward to check that function $\mathcal{E}_n(p_p, p_e, p_\nu)$ has an extremum inside the triangular region if the inequality $c_p, \ c_e > c_\nu$ and the following condition is satisfied

$$ p_n > \frac{c_\nu c_e}{(c_e^2 - c_\nu^2)^{1/2}} m_e + \frac{c_\nu c_p}{(c_p^2 - c_\nu^2)^{1/2}} m_p \ . \tag{21} $$

The total energy of the proton, electron and neutrino in this extremum is

$$ E_1(p_n) = c_\nu p_n + m_p c_p (c_p^2 - c_\nu^2)^{1/2} + m_e c_e (c_e^2 - c_\nu^2)^{1/2} \ . \tag{22} $$

Now one can readily check that the equation

$$ \sqrt{p_n^2 + m_n^2} = E_1(p_n) $$

is incompatible with the inequality (21). Therefore, in order to determine the threshold momentum p_0 of neutron stability one should find the minimum value of the function $\mathcal{E}_n(p_p, p_e, p_\nu)$ on the boundary of momentum region. For that one should consider six cases when either one or two of the momenta $p_p, \ p_e, \ p_\nu$ are zero.

Let us start with the case $p_e = 0$. The function $\mathcal{E}_n(p_p, 0, p_\nu)$ has an extremum with respect to p_p and p_ν provided that $c_p > c_\nu$ and

$$ p_n > \frac{c_\nu c_p}{(c_p^2 - c_\nu^2)^{1/2}} m_p \ . $$

The energy corresponding to this extremum is

$$ E_2(p_n) = c_\nu p_n + m_p c_p (c_p^2 - c_\nu^2)^{1/2} + m_e c_e^2 \ . $$

This energy becomes larger than the energy of neutron with the momentum p_n at very low momenta $p_n \gtrsim (m_n^2 - m_e^2)/2m_e$. Thus, this kinematical channel of the neutron decay is closed at energies higher than a few GeV even in the Lorentz-invariant case. It is straightforward to check that the same happens

11

in the case $p_p = 0$ and when any two momenta of the decay products are zero. Therefore, the minimum of interest corresponds to the case $p_\nu = 0$.

The threshold of proton instability is found in a similar way. The result is the following: in order to find the energy E_1, where proton becomes unstable one should set $p_\nu = 0$ and study two-body decay $p \to n, e$. To illustrate some differences with the case of neutron decay, let us consider just one alternative kinematical regime, corresponding to point on the boundary of the region determined by (15) where only the proton momentum is non-zero. In this case equation (13) reduces to

$$c_p\sqrt{p_1^2 + m_p^2 c_p^2} = \sqrt{p_n^2 + m_n^2} + m_e^2 c_e^4$$

which gives

$$p_1 \approx \frac{2m_e}{c_p^2 - 1}$$

This value is indeed much larger than the one in the extremum with non-zero p_e, see Eq. (19) and Fig. 1.

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